1901001101030001
EXAMINATION FEBRUARY-MARCH 2024
MASTER OF ARTS EXTERNAL PART-1
MATHEMATICS
TOPOLOGY - LEVEL 3

[Time: As Per Schedule] **[Max. Marks:100]**

Instructions:
1. Fill up strictly the following details on your answer book
   a. Name of the Examination : **MASTER OF ARTS EXTERNAL PART-1**
   b. Name of the Subject : **MATHEMATICS TOPOLOGY LEVEL 3**
   c. Subject Code No : **1901001101030001**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual notations and conventions.

Seat No:

Student's Signature

Q.1 (1) Define nowhere dense set. Show that a subset A of a topological space is nowhere dense if and only if every non-empty open set has a non-empty open subset disjoint from A. **7**

(2) Show that every second countable space is first countable. **7**

(3) Define separable space. Prove that every separable metric space is second countable. **6**

OR

(1) Define dense set and nowhere dense set. Show that a subset of a topological space is dense if and only if it intersects every non-empty open set. **7**

(2) Define closure of a set. Show that a set A is closed if and only if $A = \overline{A}$. Also, deduce that $\overline{A \cup B} = \overline{A} \cup \overline{B}$. Is $\overline{A \cap B} = \overline{A} \cap \overline{B}$? justify. **7**

(3) Let X be a topological space and A be the subset of X then prove that: **6**
(i) $\overline{A^c} = [Int\ (A)]^c$
(ii) A is closed if and only if $D\ (A) \subset\ A$.

Q.2 (1) Prove that every closed and bounded subspace of the real line is compact. **7**

(2) Let X_1 and X_2 be two topological spaces. Show that the product topology on $X = X_1 \times X_2$ is the weak topology generated by projections. **7**

(3) Define compact space. Show that a continuous real function f defined on a compact space X attains its infimum and its supremum. **6**

OR

(1) State finite intersection property. If a topological space X is compact then prove that every class of closed sets with finite intersection property has non empty intersection. **7**

(2) Prove that any continuous mapping of a compact metric space into a metric space is uniformly continuous. **7**

(3) Define countably compact space. Prove that a second countable space is countably compact if and only if it is compact. **6**

Q.3 (1) State and prove Tychonoff's theorem. **7**

(2) Prove that every compact metric space has the Bolzano-Weierstrass property. **7**

(3) Define Lebesgue number. Prove that every sequentially compact metric space is compact. **6**

OR

(1) Prove that a completely regular space is regular. **7**

(2) Define normal space. Show that a closed subspace of a normal space is normal. **7**

(3) Prove that a topological space is a T_1 space if and only if each point is a closed set. **6**

Q.4 (1) Prove that the product of any non-empty class of connected space is connected. **7**

(2) Let X be a topological space. If {Ai} is a non-empty class of connected subspace of X such that $\bigcap_i A_i$ is non-empty, then prove that $A = U_i A_i$ is also connected subspace of X. **7**

(3) Prove that the closure of connected set is connected. **6**

OR

(1) Prove that the product of any non-empty class of locally connected space is locally connected. **7**

(2) Show that R^n and C^n are connected. **7**

(3) Define locally connected space. Let X be a locally connected space. If Y is an open subspace of X, show that each component of Y is open in X. In particular, each component of X is open. **6**

Q.5 (1) Show that a compact locally connected space has a finite number of components. **7**

(2) Prove that the topological space X is locally connected if the components of every open subspace of X are open in X. **7**

(3) Prove that a subspace of a real line R is connected if and only if it is an interval. **6**

OR

(1) Prove that the product of any non-empty class of totally disconnected space is disconnected. **7**

(2) Define component of a topological space and show that each component of a topological space is closed. **7**

(3) Prove that the topological space X is locally connected if the components of every open subspace of X are open in X. **6**

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